Mathematical Modeling of COVID-19 Coronavirus

Outline and Topic Proposal

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Coronavirus disease (COVID-19) is an infectious disease caused by a new virus SARS-COV-2. The disease causes respiratory illness (like the flu) with symptoms such as a cough, fever, and in more severe cases, difficulty breathing. To date more than 20,35,000 people from all over the world have been infected while the disease has taken more than 1,35,200 lives till now. These numbers go on increasing day-by-day. Apart from this, the COVID-19 coronavirus pandemic has plunged the system into a deep crisis. The stock markets are plummeting, a recession seems inevitable. The outbreak of this pandemic is unpredictable in terms of global consequences. This paper presents a mathematical model in order to predict the epidemic patterns of this virus. The model is based on *susceptible-exposed-infected-recovered* (SEIR) family of compartmental models. SEIR epidemiological models include quarantine and isolation to study the control and intervention of this pandemic. The paper also discusses how social measures influence the parameters of the model which may change the mortality rates as well as active contaminated cases over time.

1 Scope of Work

In this section the essence of the proposed work is described by answering four key questions.

What is the problem you want to address in your work? More recently, mathematical models have been used to investigate how to more effectively predict and control various epidemic diseases with control measures including vaccination, quarantine, and isolation.

Modeling of the COVID-19 outbreak typically falls into analytically tractable models like the SEIR model which is capable of capturing some globally important phenomena like the rate of spread of diseases, distancing, regional lockdowns, quarantine and global public health vigilance using few parameters.

Why is it a problem? From a strategic and healthcare management perspective, the propagation pattern of the disease and the prediction of its spread over time is of great importance, to save lives and to minimize the social and economic consequences of the disease.

What is the solution you developed in your work? Here, we discuss the problem by considering a simple ODE model that is a commonly used SEIR-type model. In a standard SEIR model, the whole population N is divided into four sub-classes: susceptible (S), exposed but not yet infectious (E), infectious (I), and immune or recovered (R) individuals.

Why is it a solution? In fact, the main meaning of the research of infectious disease dynamics is to make people more comprehensively and deeply understand the epidemic regularity of infectious disease; then more effective control strategies are adopted to provide better theoretical support for the prevention and control of epidemics. To this end, many mathematical biology workers considered more realistic factors in the course of the study, such as population size change,migration, cross infection, and other practical factors. In the course of epidemics and outbreaks of infectious diseases, people always take various measures to control the epidemic in order to minimize the harm of epidemic diseases. Quarantine is one of the important means to prevent and control epidemic diseases.

It should be highlighted that mathematical models applied to real-world systems (social, biological, economical, etc.) are only valid under their assumptions and hypothesis. Therefore, this research— and similar ones— that address epidemic patterns, do not convey direct clinical information and dangers for the public, but should rather be used by healthcare strategists for better planning and decision making. Hence, the study of this work is only recommended for researchers familiar with the strength points and limitations of mathematical modeling of biological systems.

2 Preliminary Table of Contents

In this section the table of contents for the proposed work is described.

1. **Compartmental Modeling & Mathematical Epidemiology**
   a) **Mathematical Modeling**

A model is a conceptual or mathematical and simplified representation of a system which is usually used to understand and quantify it. It ultimately a means of systematizing the available knowledge and understanding of a given phenomenon and the facts concerning it [GF08; Sam20].

Mathematical modeling is the art of translating problems from an application area into tractable mathematical formulations whose theoretical and numerical analysis provides insight, answers, and guidance useful for the originating application [GF08; Sam20].

Mathematical modeling [Sam20]

- is indispensable in many applications
- is successful in many further applications
- gives precision and direction for problem solution
- enables a thorough understanding of the system modeled
- prepares the way for better design or control of a system
- allows the efficient use of modern computing capabilities

b) **Mathematical Epidemiology**

The goal of mathematical epidemiology is first to understand the causes of a disease, then to predict its course, and finally to develop ways of controlling it, including comparisons of different possible approaches. The first step is obtaining and analyzing observed data [FXZ07; Het09; Bra17]

2. **Compartmental Modeling**

Compartmental modeling is a technique which presents mathematical modeling of infectious diseases. In this technique, the population is divided into compartments. It is assumed that the individuals in the compartment have the same characteristics. Its origin is in the early 20th century, with an important early work being that of Kermack and McKendrick in 1927 [KM27].

These models are generally mathematically investigated by using ordinary differential equations (ODEs). The models used by ODEs are deterministic, but these models are also be investigated in a stochastic framework. The stochastic framework is more realistic but also more complicated to analyze [KM27; KM32; KM33].

Compartmental models are used in order to predict the properties a disease, how it spreads, what the duration of an epidemic will be, how many people will be infected. Apart from these, the model also allows for understanding how different situations may affect the outcome of the epidemic, e.g., what the most efficient technique is for issuing a limited number of vaccines in a given population [FXZ07].

3. **SEIR model**

Here, we discuss the problem by considering a simple ODE model that is a commonly used SEIR-type model. In a standard SEIR model, the whole population

N is divided into four sub-classes: susceptible (S), exposed but not yet infectious (E), infectious (I), and immune or recovered (R) individuals. Susceptibles become exposed (latent) at the rate $\lambda_1(t)S(t) = \beta S(t)I(t)/N$ where β is the disease transmission coefficient in the absence of interventions and N is the total population size [Bra17; Sam20; Thi18; GF08; FXZ07]. Latent individuals progress to the infectious stage at a constant rate α_1 and infectious individuals recover at a constant rate δ_1.

3 Relevant Related Work

In this section, identified related work is described.

The first study of infectious disease data was found in the work of John Graunt (1620-1674) in his 1662 book "Natural and Political Observations made upon the Bills of Mortality". The "Bills of Mortality" were weekly records of numbers and causes of death in London parishes. The records, beginning in 1592 and kept continuously from 1603 on, provided the data that Graunt used. He carried out various analysis for the causes of death and finally provided a methodology to estimate the comparative risks of dying from various diseases, giving the first approach to a theory of competing risks [Bra17].

The first model in mathematical epidemiology was described for the work carried out by Daniel Bernoulli (1700-1782) on inoculation against smallpox. Smallpox was an epidemic disease in the eighteenth century.

Another valuable contribution to the understanding of infectious diseases even before there was knowledge about the disease transmission process was the knowledge obtained by study of the temporal and spatial pattern of cholera cases in the 1855 epidemic in London by John Snow, who was able to pinpoint the Broad Street water pump as the source of the infection. In 1873, William Budd was able to achieve a similar understanding of the spread of typhoid. In 1840, William Farr studied statistical returns with the goal of discovering the laws that underlie the rise and fall of epidemics.

The mathematical theory of infectious diseases pioneered by Ross, MacDonald, Kermack, McKendrick and others has played a major role in the study of the control and prevention of infectious diseases (see, for example, [Ros10]; [KM27; KM32; KM33; KM91]). More recently, mathematical models have been used to investigate how to more effectively control epidemic diseases via various disease control measures including vaccination, quarantine, and isolation (see, for example, [AM79; AAM92; Bai+75; Cho+03; FXZ07; FT00; GF08; Het00; Lip+03; Mur07; DW02]).

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